



Tuning Temperaments and Styles

by Rob Mitchell

When I came to the piano business a few years ago, I was probably of the opinion that "a tuning was just a tuning". I imagined that the pitches (frequencies) of the various notes would be dictated by mathematical formulas (the frequency doubles every octave, right?) and that all pianos would be tuned identically. The truth could not be further from this position.

In this article, I'll start with a bit of history on temperaments¹ and then explore some of the tuning possibilities on pianos.

Historical Temperaments

The Greek philosopher Pythagoras is credited with figuring out the pitch relationships of intervals that most of us find musically pleasing:

Octave - 2 to 1

Perfect 5th - 3 to 2

Perfect 4th - 4 to 3

Major 3rd - 5 to 4

and the list goes on. So for example if A4 is 440Hz and E5 is a perfect 5th above, then the pitch for E5 should be 660Hz. However, this quickly leads to a major stumbling block: if someone goes all the way around the circle-of-fifths starting at A4 (and dropping down an octave when needed), then you overshoot A4 by about 6Hz (or about 24 cents). This is known as the "Comma of Pythagoras".

Therefore, if we tune any instrument using a Pythagorean Temperament then the music played in some key signatures will sound wonderful, while the music in other keys will sound very dissonant. Of course, this was not a major problem with instruments having only a few strings like guitars and violins. But this was a huge issue once pianos came on the scene since it is not feasible to retune for music in different keys.

Musicians wrestled with this problem for hundreds of years, trying different compromises that would sound pleasing for largest number of musical pieces. Temperaments referred to as "Just", "1/4 Comma Mean Tone" and "Well" along with countless variations were experimented with. Theoreticians have spent lifetimes exploring these concepts and this

¹ A "temperament" is the style used to place the 12 notes in a single octave. Most of the notes are "tempered" from some idealized placement.

was just intended to give a brief introduction to the topic. Suffice to say that temperaments are a complex subject and all musicians should be aware of their history.

Around the mid-19th century, musicians began to settle on what was called Equal Temperament. It is perhaps the simplest of all temperaments since it divides every octave into exactly 12 equal steps. A piece played in one key and then transposed to another should sound pretty much the same. It makes tradeoffs between slow beating intervals (the 4ths and 5ths) and rapid beating intervals (e.g. the major 3rds and 6ths). All the intervals except the 5th (and minor thirds) end up tuned slightly wide of beatless. The fifth and minor third are tuned narrow of beatless.

When playing in Equal Temperament, you may not even be aware that the different intervals are beating at different speeds. Most of us have become so accustomed to this tuning method that we don't even notice. In fact, listening to other historical styles will sound very odd at first (though I really encourage you to attend a performance if you ever have the opportunity to hear other temperaments).

Equal Temperament is used almost exclusively today, with the occasional exception of someone specifically interested in historical temperaments and music. While it would be unusual for a pianist to want something besides Equal Temperament, there are other possibilities. Ask your piano technician if you would like to try experimenting with historical temperaments.

Inharmonicity

Unfortunately for pianos, things get a bit more complicated. For all stringed instruments, when you play a note, you actually get a whole series of tones called the harmonic series. If f is the frequency of the note played, then the actual series of tones produced is:

$$f, 2f, 3f, 4f, 5f, \text{ etc.}$$

Except for pianos... Piano wire is exceptionally stiff. Because the termination points at each end of the speaking length (the agraffe and bridge in a grand) are not perfect bearings, the effective speaking length gets slightly shorter for each higher harmonic

multiple. And as a result, each harmonic is slightly sharp of where it theoretically should be.

The piano is considered an inharmonic instrument. So, on a piano, we don't even call them "harmonics" but instead: Partial. The fundamental at f is the "first partial", slightly sharp of $2f$ is the "second partial", and so on.

Generally speaking, bigger pianos with longer strings have lower inharmonicity (that is, the actual pitch is closer to the theoretical value). Large grands are usually known for having very low inharmonicity while small spinets typically have very high inharmonicity.

Here's an example. On a Steinway L (a 5' 11" grand that would be considered to have low to moderate inharmonicity) I deliberately tuned A2 to 110Hz and measured the pitch of all the partials. Here are the results (frequencies are in hertz):

partial #	1	2	3	4	5	6	7	8
note	A2	A3	E4	A4	C#5	E5	G5	A5
ideal freq	110	220	330	440	550	660	770	880
actual freq	110.0	220.6	330.8	441.4	552.4	663.4	774.8	886.8
		↑		↑		↑		↑
		2:1		4:2		6:3		8:4

(Across the bottom, I've shown where the partials are located for different styles of the A2-A3 octave.)

Inharmonicity causes the most noticeable problems when it comes to tuning octaves. Ideally with an octave interval, the higher note is exactly twice the frequency of the lower and all the higher partials "line up" with each other. But because of inharmonicity, it turns out that it's not possible to exactly align any more than one pair of partials at a time, though a piano tuner can get much closer with a grand than with a spinet.

Technicians have developed a terminology for describing the types of octaves they are tuning. For a "2:1 octave" the first partial of the higher note would be tuned at exactly the second partial of the lower. Tuning the A2 - A3 octave from above, A3 would be tuned to 220.6Hz. For a "4:2 octave" (at A4 in the table above) the fourth partial of lower note (or two octaves up) would be aligned with the second partial of the higher (one octave up). A3 would be placed a bit sharper from the 220.6Hz that was correct for the 2:1 octave (the exact placement would depend on the location of the partials for A3). The basic idea here is to make partials lining up at 4:2 sound good without compromising the 2:1 partials too much.

The tunings of the octaves get increasingly "wider" as we go up in numbers. So if an octave were tuned as 4:2, then a very slow roll will be heard at the 2:1 partials since it would be slightly on the wide side (that is the higher note in the octave would be slightly sharp).

Octave Tuning Styles

The preferred style or amount of stretch in the octave varies with different parts of the keyboard. Starting in the high treble, most pianists and technicians generally like 2:1 octaves. The notes are so high and the partials even higher in pitch that most of us can't hear whether the upper partials are matched.

Moving down the keyboard, most listeners will agree that 4:2 octaves start sounding more pleasing than 2:1. This usually happens around the fifth or sixth octave. Further down and 6:3 octaves start sounding better, usually somewhere in the third octave. Going all the way down to the deep bass and 8:4, 10:5 or even 12:6 octaves can sound great on the biggest grands.

But all of this is fairly subjective and a matter of personal taste. For smaller spinets and uprights, your technician will generally do the best they can to produce pleasant sounding octaves. Since the inharmonicity will probably be fairly high, it's usually best to keep the octaves on the narrow side to at least get good 2:1 sounding octave intervals.

For larger, higher quality grands, the piano owner has more flexibility in how much stretch they prefer. To anyone who has their piano tuned on a regular basis, I would encourage you to experiment with different amounts of stretch and different kinds of music. Try a narrow octave tuning one time and then a wider tuning the next just to see what you prefer. A multi-voice fugue will sound very different from a big bombastic piece with lots of chords and arpeggios in either stretch style.

Summary

There are variations in both temperament and octave stretch that available to the pianist. Historical temperaments will sound very different to anyone accustomed to Equal Temperament (i.e. most of us). Differences in octave stretch are more subtle but can produce variations in sound depending on the size of the piano and the kinds of music being played.

For more information or to request an appointment, visit www.mitchellpianoservice.com.